The Political Economy of Conservation Buy-Outs

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Abstract

This paper characterizes how the combination of top-down (regulatory) and bottom-up (voluntary) approaches to curbing externalities can affect welfare when both are imperfect. On the one hand, private provision of many public goods is quite common but likely far short of efficient. On the other hand, there is reason to doubt that existing price and quantity instruments are calibrated optimally. Given the coexistence of these contrasting approaches, under what conditions does allowing private provision of public goods on top of regulatory instruments improve welfare? The traditional comparison of top-down price and quantity instruments assumes no voluntary reductions in externality-generating behaviors, but recent research demonstrates that such voluntary reductions change the characterization of efficient regulatory prices and quantities. We build on this framework and characterize the welfare implications of various combinations of price instruments, quantity instruments, and voluntary provision. We find that allowing private parties to pursue voluntary reductions increases welfare except in the presence of a relatively stringent Pigouvian tax.

1. Introduction

Market-based approaches to environmental regulation have become increasingly salient among both policymakers and academics. Whereas traditional command-and-control regulations mandate the use of specific technologies, inputs, or processes to reduce externalities resulting from economically beneficial activity, market-based approaches in the tradition of Pigou (1932) establish a "price" (a fee or tax) for externalities such as pollution or a "cap" (a permissible quantity) and leverage firms' profit-maximizing behavior to ensure cost-effective reductions in externality-producing activities.

Historically, proponents of Pigouvian price or quantity controls have evaluated their performance under the assumption that voluntary actions to reduce externalities do not occur or would be negligible (e.g., Weitzman (1974); Oates and Baumol (1975)). In reality, voluntary efforts to curb externalities ranging from local emissions to global CO2 concentrations are widespread and growing in prevalence. In recognition of this fact, recent work has revisited the analysis of "price vs. quantities" and how to optimally set either instrument in the presence of non-trivial voluntary efforts, termed "Coasean provision" (Costello and Kotchen, 2020; Chan, 2022).

In this paper, we leverage these recent advances to study the impacts of Coasean provision in the context of *natural resource* governance. Market-based approaches have also become prevalent in the governance of natural resources subject to common-pool problems such as surface water, groundwater and fisheries (Leonard, Costello, and Libecap, 2020). Natural resource governance has (at least) two features that distinguish it from the regulation of many externalities. First, many publicly managed natural resources are subject to *both* price *and* quantity regulation. For instance, the Bureau of Land Management determines the total number of acres available for oil and gas drilling on public lands (the quantity) and also charges a royalty on extracted products (the fee). Understanding Coasean provision in this context requires an expanded model with both prices *and* quantities.

The second distinguishing feature of natural resource governance is that Coasean provision can be (and often is) explicitly forbidden when resources are publicly owned and managed. It is infeasible to ban Coasean provision for most externalities because they tend to arise from activities that are both regulated and unregulated. For instance, greenhouse gas emissions can be affected by voluntary decisions or contractual arrangements that reduce a variety of activities ranging from automobile vehicle miles traveled to deforestation, and regulation of these voluntary reductions is infeasible. In contrast, when governments own natural resources and set the terms of their use, they determine who is eligible to acquire rights to those resources (e.g., drilling permits, water rights, or individual fishing quota). The upshot is that unlike for many externalities, for natural resources, whether to allow Coasean provision is itself a policy parameter.

In this paper, we extend the approach of Costello and Kotchen (2020) to build a model of prices *and* quantities with Coasean provision to determine whether and when allowing Coasean provision is likely to increase welfare. Leonard et al. (2021) point out that Coasean provision is currently illegal for most publicly managed resources in the United States (and globally) and argue that environmentalists should be to able to acquire "non-use rights" to these resources if they wish to advance conservation beyond the regulatory baseline (e.g., the cap). There has also been increasing interest in "supply side" approaches to climate policy that focus on buying up fossil fuel deposits to prevent extraction Harstad (2012); Asheim et al. (2019). Our goal is to provide a general characterization of the conditions under which approaches like these are likely to improve welfare. Whereas Costello and Kotchen (2020) consider optimal instrument choice given the presence of Coasean provision, we ask how introducing Coasean provision will affect welfare for a given mix of policy instruments, regardless of whether these instruments are set optimally.

We find that the welfare implications of Coasean provision depend critically on both the cap and the fee in an interesting way. In the absence of Coasean provision, only the cap is relevant for overall welfare because it dictates the level of extraction, while the fee determines transfers from resource users to taxpayers.[1] In the presence of Coasean provision, only the *fee* is relevant for welfare because it acts as a subsidy to environmental buyers and determines the equilibrium level of extraction. Importantly, Coasean provision can never improve welfare

[1]This assumes that the cap is the "binding" policy instrument in the absence of Coasean provision. As we discuss in Section 2, the case where the fee is the binding policy instrument is equivalent to a setting with solely a fee and no cap.

if the existing mix of policy instruments is either efficient or overly stringent—Coasean provision can only help when existing policy is too lax.

Our analysis proceeds in several steps. In Section 2, we introduce the basic "prices and quantities" in the context of a summary of several key publicly managed resources in the US. This provides a baseline for thinking about the potential implications of Coasean provision. An advantage of this model is that it nests as special cases instances where resources are subject only to a quantity cap (such as many fisheries) or only to a price.

In Section 3, we develop our core theoretical results on the welfare implications of Coasean provision. First, we characterize four classes of possible outcomes from allowing Coasean provision. Second, we derive necessary and sufficient conditions for Coasean provision to be welfare improving. These conditions are expressed in terms of the initial cap and fee, and the extent of Coasean provision (i.e. the degree of free-riding), expressed as a proportion of the overall marginal social cost of resource use. Third, we consider the separate and joint roles of both free-riding behavior and fixed transaction costs in affecting the welfare effects of Coasean provision. We conclude this section with a set of simplified policy punchlines.

2. Background and Model Setup

2.1. Natural Resource Governance with Prices and Quantities

Many natural resource stocks are owned and managed by governments. One reason for this is widespread government ownership of land. In the United States the federal government owns roughly 30% of the land, including about half the land in the American West. As a result, the government manages considerable oil, gas, and mineral deposits, timber, and grazing grazing resources across agencies including the Bureau of Land Management, the U.S. Forest Services, and various state governments. In addition, federal and state governments manage surface water and marine ecosystems. Globally, most governments retain ownership and management of subsurface resources, even when land is privatized (unlike in the United States). Finally, externality-related cap-and-trade markets can be thought of within this framework, to the extent that the government initially owns all the pollution permits it creates before allocating them to firms.

Our model builds upon the structure outlined in Costello and Kotchen (2020) and is purposefully simplified and abstracted from specific applications so as to illustrate the general principles at play. The extraction of publicly owned natural resources generates a litany of private and external costs and benefits that governments must weigh when setting policy. Suppose one group of individuals and/or corporations (i.e. "the resource user") derives benefits through the degradation or consumptive use of a public natural resource Q. The benefits they obtain from these activities $B(Q)$ are increasing in Q at a weakly decreasing rate, such that $B'(Q) = MB(Q)$ is (weakly) downward sloping. This is what a private extractor would be willing to pay for another unit of resource extraction.

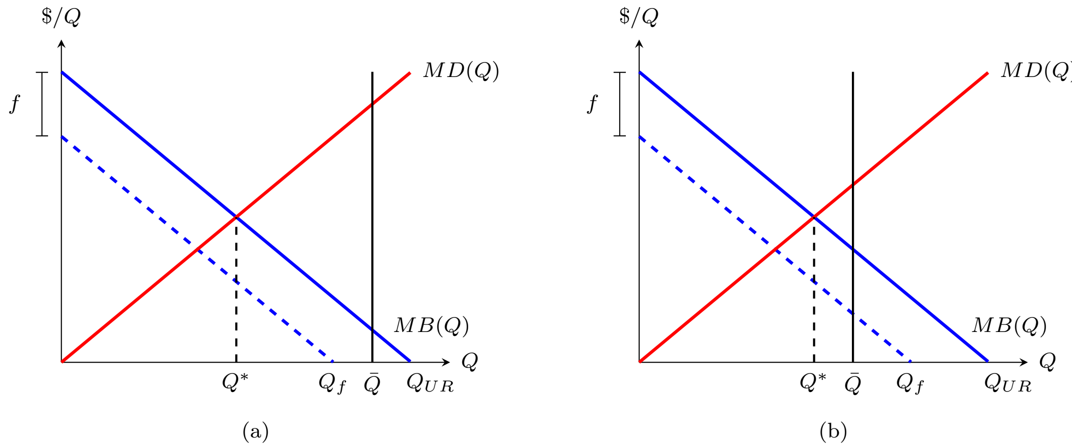
We also assume that extraction entails costs or damages that are "public" (i.e. non-rival and non-exclusive) in nature to a group of citizens $i = 1, ..., N$ (i.e. "the public"), such that the aggregate marginal damages to citizens can be written as the vertical sum of individual marginal damages, $MD(Q) = \sum_{i=1}^{N} MD_i(Q)$. We assume aggregate marginal damages are increasing in Q.[2] These damages will vary by resource and could be driven by a variety of factors. For some resources and activities such as fossil fuel extraction and livestock grazing, $MD(Q)$ includes climate-related damages. For other resources such as timber and water, there may be important ecosystem and biodiversity implications of extraction. Resources and the landscapes in which they are found may also entail existence values that are lost via resource production (Krutilla, 1967; Krutilla et al., 1983).

The optimal balance of these social costs and private benefits that maximizes net benefits to society occurs at Q^*, where $MB(Q^*) = MD(Q^*)$, depicted in panel (a) of Figure 1.[3] Governments face a variety of options for determining the actual level of extraction of the resources which they own. One possibility is to charge a fee f per unit of extraction. Doing so reduces the private demand for extraction. The classic result from Pigou (1932) is that setting f equal to $MD(Q^*)$ would result in the efficient level of extraction. In practice, governments often set fees quite low, as depicted in panel (b) of Figure 1. For the fee

[2]Importantly, these damages must be exogenous to citizens in that they cannot engage in private abatement activity that would influence the magnitude of damages (Costello and Kotchen, 2020).

[3]This depends on the existence of an interior solution. Alternatively, if $MB(Q) < MD(Q)\ \forall Q$ then the optimal solution is a corner solution where $Q = 0$.

Figure 1: Resource Governance with Prices and Quantities



Notes: This figure depicts the management of natural resources with a combination of prices (fees) and quantities (extraction caps). Panel (a) depicts the efficient level of extraction. Panel (b) depicts management with a fee f only. Panel (c) depicts an example of both a cap $\bar{Q}$ and a fee f where the fee is the binding instrument and the cap is irrelevant. Panel (d) depicts a case where the cap $\bar{Q}$ is the binding instrument but extractors also pay a fee f for each unit of the resource.

depicted here, extractors would choose to utilize Q^f units of the resource, which exceeds the efficient level Q^*. The standard alternative to setting a price f for resource use is to cap total extraction at some level $\bar{Q}$. In this case, total extraction will equal $\bar{Q}$ as long as $MB(\bar{Q}) \geq 0$. Efficiency would require $\bar{Q} = Q^*$, but again, many resources may be subject to more lax extraction caps.

Although the literature in environmental economics has tended focus on the comparison of price *or* quantities (Weitzman, 1974; Costello and Kotchen, 2020), many natural resources are actually governed by a combination of the two instruments. Most government-owned resource stocks are subject to a two-part process whereby policymakers first decide how much of the resource to "open" for extraction ($\bar{Q}$), and then charge a fee for extraction privileges (f). These fees can take a variety of forms ranging from a per-unit fee (grazing), to a royalty percentage of gross revenue (oil), to a simple agency cost recovery fee (many fisheries). Fees also serve a variety purposes. Oil and gas royalties on federal leases help to fund the Land and Water Conservation Fund, whereas the nominal fees assessed in fisheries simply cover administrative costs.

The combination of both prices and quantities in the governance of public natural resources leads to two possible policy regimes. In the first regime, the cap is lax enough relative to the fee that the fee is the binding instrument that determines extraction, such that total extraction demand is less than the quantity opened for extraction: $Q_f < \bar{Q}$. This case is analytically identical to the fee-only regime depicted in panel (b) because the cap $\bar{Q}$ becomes meaningless for determining behavior. This case, depicted in panel (c) of Figure 1, will not play a major role in our analysis.

The other possible "prices and quantities" regime is depicted in panel (d) of Figure 1. Here, the resource cap $\bar{Q}$ is the binding instrument. In this scenario, extractors are still willing to pay $MB(Q) - f$ for every unit of the resource available. In this case, the price f does not affect extraction Q, and so only matters for determining the level of transfers from extractors to the government (or taxpayers), but not for total welfare. This will be the case as long as extractors have a positive willingness to pay at $\bar{Q}$, or equivalently, if $MB(\bar{Q}) > f$. As we show below, allowing Coasean provision under this policy regime makes both the cap *and* the fee relevant for welfare.

Throughout this paper, our treatment of price and quantity instruments, and their combination, assumes that the binding instrument is unlikely to be set at an efficient level. Political economy considerations in resource management, including revenue-raising requirements, may lead to fees or quotas being set at levels that yield pollution levels that are often too high, although it is possible that fees or quotas may be set too stringently as well. In general, the legislation that constrains regulatory actions often requires managers to consider multiple objectives beyond efficiency, suggesting there is little reason that price and quantity policies will achieve Q^*. Therefore, this opens up the possibility that allowing voluntary transactions between resource users and individuals or interest groups *on top of* existing regulatory strictures could improve societal welfare.

2.2. Coasean Provision and "Non-Use Rights"

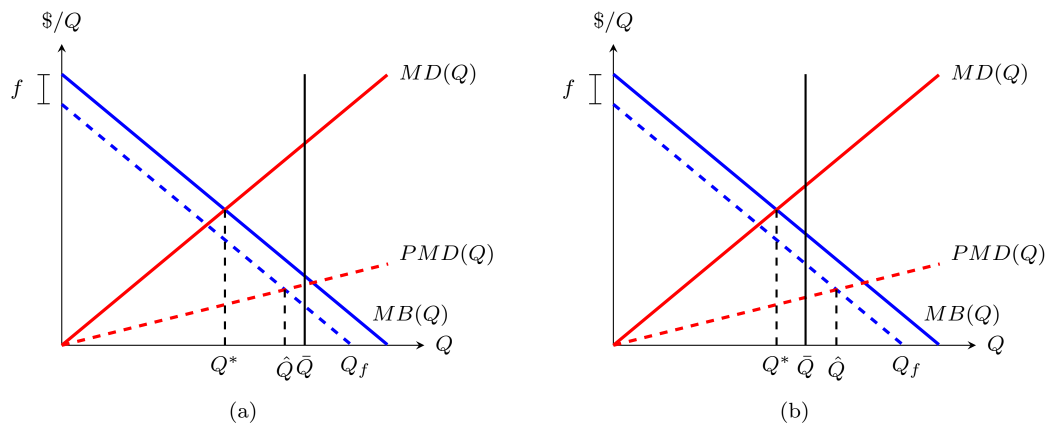
The variety in the scope, nature, and degree of services generated by public lands and natural resources suggest the possibility for substantial heterogeneity in the value ascribed to these resources by individuals. Increasingly, private individuals, environmental non-governmental organizations (ENGOs) and other groups such as sportsman's associations have demonstrated a willingness to voluntarily contribute to the public goods associated with natural resource conservation (Anderson and Parker, 2013; Costello and Kotchen, 2020; Leonard, Costello, and Libecap, 2020). In the context of government-managed natural resources, conservation could, in theory, be achieved directly if ENGOs or individual purchased resource rights from extractors and held those rights rather than exercising them.

In practice, it is often illegal for ENGOs and individuals to acquire rights to government-owned natural resources for non-extractive purposes because most resources in the United States are government by laws dating back to the nineteenth century (Leonard and Regan, 2019; Leonard et al., 2021). These laws contain "use-it-or-lose-it" provisions designed to prevent speculation and monopolization during Westward Expansion. Given the mismatch between these nineteenth century policies and modern resource demands and conservation values, Leonard et al. (2021) advocate for "non-use" rights that would allow conservationists to participate in natural resource markets. This participation in analogous to the private "Coasean provision" of environmental quality analyzed by Costello and Kotchen (2020).

Here, we provide a brief example of how we conceptualize Coasean provision in the prices-and-quantities model governing most natural resources. The particular values of $\bar{Q}$ and f chosen by the regulator define the initial property rights of the resource user and the public—the resource users' entitlements to the use of natural resources and the public's rights to a level of environmental quality, respectively. However, this initial assignment of rights may not reflect the final assignment if approaches exist, with sufficiently low transaction costs, for members of the public to coordinate and aggregate their collective WTP for additional environmental quality in order to compensate resource users to extract less. In practice, the process of converting potential demand from the public into realized demand is complex and fraught with incentive challenges, most notably a tendency to free ride given the non-excludable nature of the benefits from reducing Q. Therefore, the ultimate demand for abatement of resource use depends on the realized *private* marginal damages from Q, $PMD(Q)$, rather than social marginal benefits, where $PMD(Q) \leq MD(Q)$.

In practice, NGOs or private sector environmental entrepreneurs (Anderson and Parker, 2013) will play a pivotal role in actualizing this private demand by overcoming the challenges of free riding in contributions and in providing a single entity with which resource users can

Figure 2: Coasean Provision with Price and Quantity Instruments



Notes: This figure depicts an example of how Coasean Provision would impact overall resource use. $\hat{Q}$ denotes the level of actual resource extraction after ENGOs purchase $\bar{Q} - \hat{Q}$ extraction rights from firms.

bargain. Later in the paper we will find it useful at times to view private marginal damages in terms of the fraction of potential demand that is actualized β, where $0 \leq \beta \leq 1$. Therefore $PMD(Q) = \beta MD(Q)$. In general, there is no reason to expect that β must be a constant, regardless of the level of resource use (i.e. the current level of environmental quality); the fraction of potential demand that is aggregated by ENGOs and other actors may vary based on Q so that $PMD(Q) = \beta(Q)MD(Q)$. Nevertheless, we assume that the behavior of $\beta(Q)$ is such that $PMD(Q)$ is always (weakly) increasing in Q.[4] This assumption is sufficient (albeit not necessary) to rule out multiple equilibria under Coasean provision.

Our analysis is based on the assumption that the resource user must pay the fee f for every unit of resource they use or pollution they produce. Furthermore, if they decide to utilize less resource or produce less pollution due to Coasean bargaining than their baseline level under the regulation, then they do not (and, unless assumed otherwise below, the ENGO does not) have to pay the fee for any of this "abatement." Given these assumptions, the marginal willingness to accept (WTA) compensation is $MB(Q) - f$. This is because resource users must be paid at least as much as the marginal net benefit they would receive

[4]This condition is guaranteed so long as $\beta' MD(Q) + \beta MD'(Q) \geq 0$, which holds if $\epsilon_{\beta,Q} + \epsilon_{MD,Q} \geq 0$, where $\epsilon_{\beta,Q}$ and $\epsilon_{MD,Q}$ are the elasticity of β and social marginal damages with respect to resource use, respectively. Therefore, the contribution share β can actually decrease in Q so long as it is less responsive, in a percentage sense, than social marginal damages.

from another unit of Q; this is comprised of the additional profits arising from that unit of the resource $MB(Q)$ minus the incremental cost f.

If Coasean provision is allowed, ENGOs will purchase extraction rights as long as $PMD(Q) > MB(Q) - f$. Hence, the equilibrium level of extraction under Coasean provision will be $\hat{Q}$, where $PMD(\hat{Q}) \equiv MB(\hat{Q}) - f$, as depicted in Figure 2. In the example depicted, ENGOs willingness to pay to reduce extraction at $\bar{Q}$ exceeds the marginal benefit of extraction, and so extraction rights in the amount $(\bar{Q} - \hat{Q})$ would be sold from extractors to ENGOs. Before proceeding to a more formal analysis of the welfare implications of Coasean provision, we emphasize two points. First, the fee f acts as an implicit subsidy to ENGOs by lowering extractors' willingness to accept payments. Second, the cap $\bar{Q}$ is no longer relevant for the total level of extraction (and therefore, welfare) if Coasean provision is allowed.

3. The Welfare Implications of Coasean Provision

3.1. Typology of Possible outcomes

Given some initial cap $\bar{Q}$ and fee f, how does allowing Coasean provision affect welfare? To understand how allowing Coasean provision is influenced by the interaction of $(\bar{Q}, f)$ it is useful to parse the potential equilibria into the following qualitatively distinct categories:

1. *Welfare irrelevant*
2. *Welfare reducing*
3. *Welfare improving*
4. *Welfare ambiguous*

These cases are presented in greater detail in Appendix A.

In the first ***welfare irrelevant*** case, the quota is set at a sufficiently stringent level that it precludes any additional reductions in Q from Coasean bargaining (i.e., $\bar{Q} < \hat{Q}$). Defining a "sufficiently stringent" quota, however, depends critically upon the level of fee. If $PMD(\bar{Q}) < MB(\bar{Q}) - f$ then $\bar{Q}$ will bind even if Coasean provision is allowed because $WTP(\bar{Q}) < WTA(\bar{Q})$. This defines a threshold for a relevant f that decreases as $\bar{Q}$ increases. In essence, the subsidy to Coasean provision from the fee is insufficient to induce any additional provision by Coasean means. When f falls below this threshold, allowing Coasean provision is irrelevant for welfare outcomes. Note, that there is nothing inherent about the level of $\bar{Q}$ alone that guarantees this outcome. In general any $\bar{Q}$, no matter how small, is consistent with additional Coasean provision so long as the fee is large enough to overcome the gap between the pre-fee MWTA of resource users and the realized MWTP of the NGO (i.e. $f > MB(\bar{Q}) - PMD(\bar{Q})$). The *combination* of the fee and quota determines whether allowing Coasean provisioning is relevant or not.[5]

The second case corresponds to a range of $(\bar{Q}, f)$ where Coasean provision is unambiguously ***welfare-reducing***. This occurs where the no-CP regulatory equilibrium is already less than or equal to the efficient level, $min(\bar{Q}, Q_f) \leq Q^*$, so that any further reductions from CP can only reduce welfare. This occurs when $f \geq MB(Q^*)$ or when $\bar{Q} \leq Q^*$. These scenarios are explored separately for the cap and for the fee in Costello and Kotchen (2020), who consider the impact of Coasean provision on optimal policy.

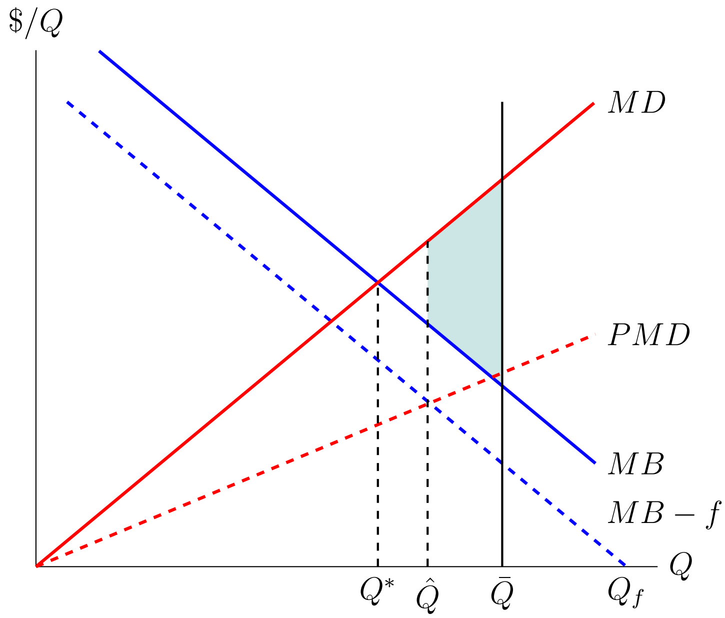
Allowing Coasean provision is unambiguously ***welfare improving*** as long as the CP equilibrium quantity is more stringent than under regulation alone *and* is greater than or equal to the efficient quantity: $Q^* \leq \hat{Q} < min(Q_f, \bar{Q})$. In this case, Coasean provision on top of the regulatory requirements moves the equilibrium resource use toward the efficient level without "overshooting" it, resulting in a clear welfare improvement. An example is depicted in Figure 3.

Coasean provision is only unambiguosly welfare-improving if the fee is large enough to induce CP but no larger than the level that would induce the efficient Q: $MB(\bar{Q}) - PMD(\bar{Q}) < f \leq MB(Q^*) - PMD(Q^*)$. In this case the magnitude of welfare gain will increase in the difference between the level of resource use under regulation alone and with Coasean provision, $min(Q_f, \bar{Q}) - \hat{Q}$.

Finally, allowing Coasean provision is ***welfare ambiguous*** when the fee and quota are such that the baseline resource use under regulation is larger than the efficient level, yet the CP equilibrium lies below Q^*: $\hat{Q} < Q^* < min(\bar{Q}, Q_f)$. This occurs when f is such that

[5] Note that if the quota is set above the no-regulation Coasean bargaining outcome, $MB(Q^{Coase}) = PMD(Q^{Coase})$, then the critical value for the fee becomes negative. This implies that quotas greater than Q^{Coase} will induce Coasean bargaining for *any* non-negative fee and therefore be welfare relevant.

Figure 3: Welfare-Improving Coasean Provision



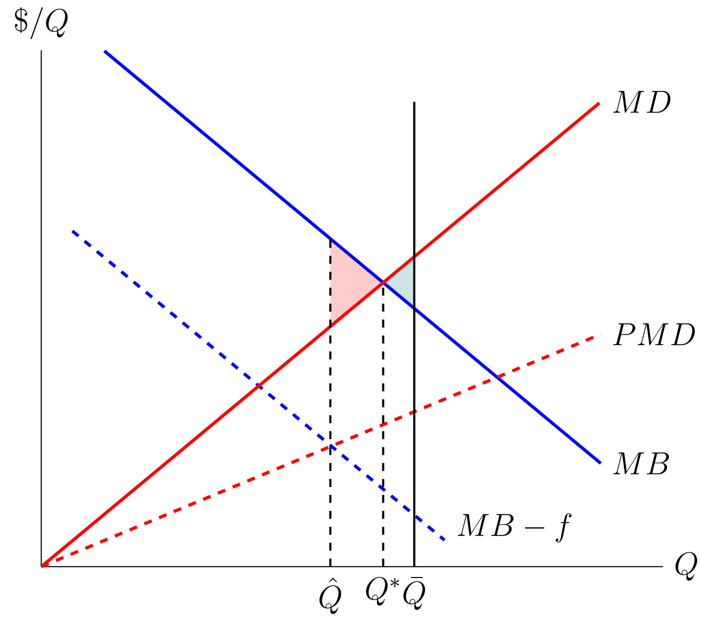
Notes: This figure depicts an example of when Coasean Provision would be unambiguously *welfare improving* because $Q^* \leq \hat{Q} < min(Q_f, \bar{Q})$.

$MB(Q^*) - PMD(Q^*) < f < MB(Q^*)$ and $\bar{Q} > Q^*$. In this case there is an "overshoot" of the efficient equilibrium, such that there are welfare gains from some units of Coasean provision and losses for others.

Figure 4 depcits an example where the net effect is a reduction in welfare from CP. Whether the net benefits of CP are positive or negative in these cases depends in general upon the shapes of the MB and MD curves on either side of the efficient quantity and the size of the gap between the regulated quantity and the efficient quantity—$min(\bar{Q}, Q_f) - Q^*$, which dictates the size of the positive welfare triangle—versus the gap between the efficient quantity and the CP equilibrium, $Q^* - \hat{Q}$. In general, for a given f, welfare gains (losses) will be at their largest (least negative) when the quota is set at the largest level that binds under the no-CP equilibrium, $\bar{Q} = Q_f$.

To summarize, the implications of allowing CP depend critically on the existing policy regime and on the expected extent of CP relative to the full social costs of extraction. It is especially worth noting that both the quota *and* fee level are relevant for determining the welfare gain from CP, since the quota level, if binding under regulation alone, determines the

Figure 4: "Overshoot" Due to Coasesian Provision



Notes: This figure depicts a case where Coasean provision, on top of quota/fee regulation, "overshoots" the optimal equilibrium. In this case the net welfare effect of Coasean provision, the green minus the pink areas, is negative.

no-CP baseline, while the fee completely determines the CP equilibrium. Next, we formally describe necessary conditions for CP to lead to welfare improvements.

3.2. Conditions for Welfare Improvements

First, it is useful to formally define the conditions under which CP will actually occur. Whether this interior Coasean provision (CP) equilibrium can be achieved or not depends crucially upon the stringency of the quota given the level of the fee, or, equivalently, whether the fee is sufficiently high to support additional Coasean provision on top of $\bar{Q}$. Specifically, if the quota is set at such a stringent level that the gap between pre-fee marginal WTA and WTP is larger than the subsidy provided by the fee, $MB(\bar{Q}) - PMD(\bar{Q}) > f$, then $\bar{Q}$ is binding both before and after allowing Coasean provision, so that allowing Coasean provision yields no additional reduction in Q. The following proposition follows.

Proposition 1. *The level of resource use under Coasean provision and the combined fee and quota* $\hat{Q}$ *is defined by* $PMD(\hat{Q}) = MB(\hat{Q}) - f$ *if* $f > MB(\bar{Q}) - PMD(\bar{Q})$*. Otherwise,*

$\hat{Q} = \bar{Q}$*, and no additional Coasean provision occurs.*

Given the assumption that $MB'(Q) < 0$ and $PMD'(Q) \geq 0$, it follows that as the stringency of the quota increases (i.e. $\bar{Q}$ decreases), the size of the minimum fee required to induce additional provision under Coasean provision increases. Furthermore, if the quota exceeds the quantity achieved under Coasean provision alone (i.e. where $f = 0$ so that $MB(Q) = PMD(Q)$), then $MB(\bar{Q}) - PMD(\bar{Q}) < 0$, implying an upper bound for a pollution *subsidy* such that Coasean provision increases environmental quality.

Assuming $(\bar{Q}, f)$ satisfy the conditions for an interior CP equilibrium given in Proposition 1, then the equilibrium condition is equivalent to that outlined for Coasean provision in the simple tax/fee Costello and Kotchen (2020). In this case the fee acts as an implicit subsidy for Coasean provision by lowering the WTA for every unit of abatement in resource use, creating incentives for reductions in Q over and above levels that would be achieved by Coasean provision in the absence of regulation. Furthermore, the influence of f and $\bar{Q}$ are asymmetric on Q

Proposition 2. *If the conditions for an interior CP equilibrium are satisfied (Proposition 1) then equilibrium CP resource use is 1) strictly decreasing in the fee* $\frac{d\hat{Q}}{df} < 0$*, and 2) invariant to changes in the quota* $\frac{d\hat{Q}}{d\bar{Q}} = 0$.

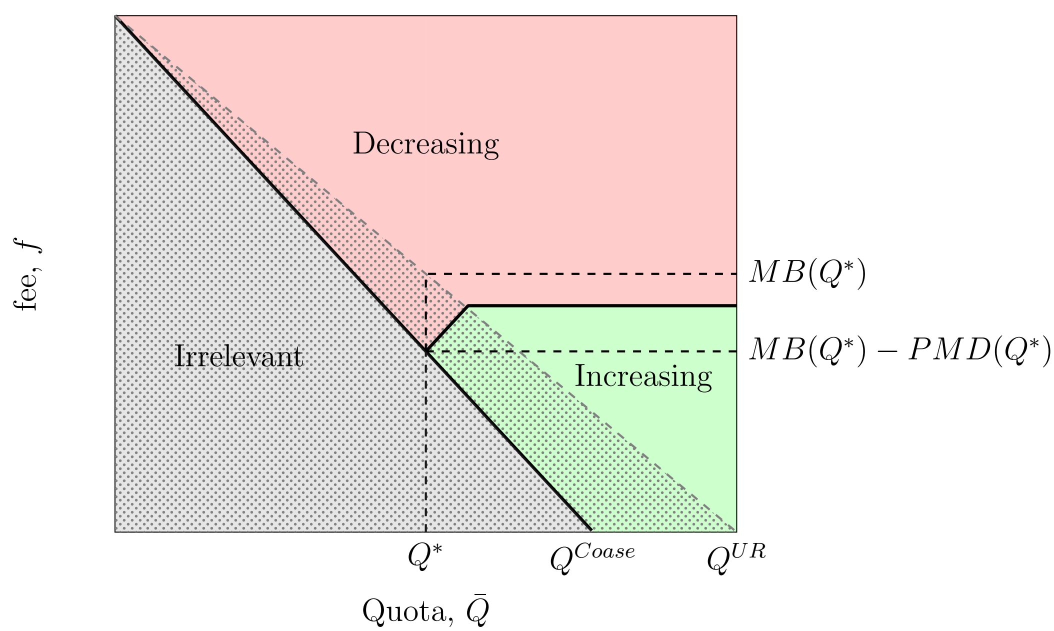
Proof. 1) The equilibrium condition under these conditions is $f = MB(\hat{Q}) - PMD(\hat{Q})$. Increasing f requires the gap between marginal benefits of Q and private marginal damages to increase, which given the slopes of these curves, necessitates a lower $\hat{Q}$. 2) The CP equilibrium condition is independent of $\bar{Q}$; therefore, $\frac{d\hat{Q}}{d\bar{Q}} = 0$ □

Proposition 2 implies that, so long as the fee is sufficiently large to engage Coasean provision, the level of the original cap/quota plays *no* role in the ultimate level of provision at the margin. Rather, the equilibrium under CP is entirely dependent on f, where the implicit subsidy to CP provided by the fee directly reduces resource use.

This finding presents an interesting dichotomy from the common case where, without Coasean provision, the quota is the binding instrument ($f < MB(\bar{Q})$), while the fee serves an ancillary revenue-raising role. In this case the level of resource use and welfare depends solely upon $\bar{Q}$, whereas once Coasean provision is allowed on top of the regulatory solution, the outcome and welfare depend entirely upon the fee, assuming the fee is sufficiently large (Proposition 1).

With these conditions established, Figure 5 shows how different combinations of $(\bar{Q}, f)$ result in increases, decreases, or no change in welfare. This figure is drawn for the special case of linear MB, MD, and PMD functions. However, as demonstrated in Appendix B, all of the qualitative properties (e.g., slopes and points of intersection) of the boundaries in the figure hold in the more general case as long as these curves are monotonic in their respective slopes and there is an interior optimal equilibrium. The division between the respective regions is distinguished by two different boundaries—one defining policy combinations where CP is irrelevant to welfare, and another defining the boundary between policies where CP decreases vs. increases welfare.

Figure 5: Welfare Implications of Coasean Provision



Notes: This figure depicts the welfare effects of Coasean provision given levels of the quota and fee with relatively high levels of free riding ($\beta = 0.3$). Q^{UR} is the unregulated equilibrium without any Coasean provision. Q^{Coase} is the Coasean bargaining equilibrium in the absence of any policy. Cross-hatching indicates regions where the quota binds prior to Coasean provision.

The green area defines combinations of $\bar{Q}$ and f that yield welfare improvements under CP. The lower bound of this region is defined by levels of fee (given the quota level) that are too low to induce Coasean provision for a given quota. This minimal fee declines—at a linear rate given linear MD and PMD curves—for quota levels up to the pure Coasean equlibrium Q^{Coase}. As noted previously, beyond this point $MB(\bar{Q}) - PMD(\bar{Q})$ is negative, so that no implicit subsidization through the fee is required to induce welfare-improving Coaseaan provision. This explains the lower bound of the increasing welfare region being $f = 0$ regardless of quota level for $\bar{Q} > Q^{Coase}$

The upper boundary for the welfare-improving region lies in the "welfare ambiguous" region defined previously: $MB(Q^*) - PMD(Q^*) < f < MB(Q^*)$. Importantly, a portion of the area adjoining the welfare-irrelevant boundary consists of values of $(\bar{Q}, f)$ such that the quota binds prior to any Coasean provision.[6] In this region it can be proven (see Appendix B) that the upper boundary for a welfare-improving fee is upward sloping in $\bar{Q}$ (Figure 5). Intuitively, as the quota increases, this expands the positive welfare triangle to the right of the efficient equilibrium (see Figure 4); therefore, to keep welfare under CP unchanged, the fee must increase so that $\hat{Q}$ decreases and the negative welfare triangle from CP "overshoot" grows to compensate.

The increasing/decreasing welfare frontier cannot slope upward indefinitely, however. Eventually the fee reaches a level where it binds in the no-CP regulatory equilibrium rather than the quota ($MB(\bar{Q}) = f$). Beyond this point, any further increases in $\bar{Q}$ have no effect on welfare under CP since both the pre and post-CP equilibria are now defined by f. As a result, the upper bound on welfare-improving fees is invariant to the quota level when the fee binds in the no-CP equilibrium.

Together, these findings yield the following:

Proposition 3. *For any $\bar{Q} \geq Q^*$ and non-zero free-riding ($MD(Q) > PMD(Q)\, \forall Q$) there is a range of fees, $f \in (f_0, f_1)$, such that welfare increases when Coasean provision is allowed.*

Proof. The proof follows immediately from the properties of the lower and upper bounds for the welfare-improving region as derived in Appendix B. □

[6] This can be seen by noting that the welfare-irrelevance region is defined by $f < MB(\bar{Q}) - PMD(\bar{Q})$, whereas quota binds when $f < MB(\bar{Q})$. The set defined by the latter inequality contains that of the former as long as free-riding isn't total (i.e. $PMD(Q) > 0\, \forall Q$).

Corollary 3.1. *The range of welfare improving fees under CP, $f \in (f_0, f_1)$, weakly decreases as $\bar{Q} \to Q^*$ from above and strongly decreases when $\bar{Q}$ approaches sufficiently close to Q^*.*

The lower bound of (f_0, f_1) will increase for any $\bar{Q} < Q^{Coase}$, whereas f_1 will decrease as $\bar{Q} \to Q^*$ once f_1 falls within the region where quota binds rather than the fee (Figure 5).

3.3. The Role of Free-Riding and Transaction Costs

3.3.1. The effect of free-riding

Figure 6 illustrates the effects of a reduction in free-riding by repeating the setting in Figure 5 but raising the value of β from 0.3 to 0.7. Several changes occur as a result.

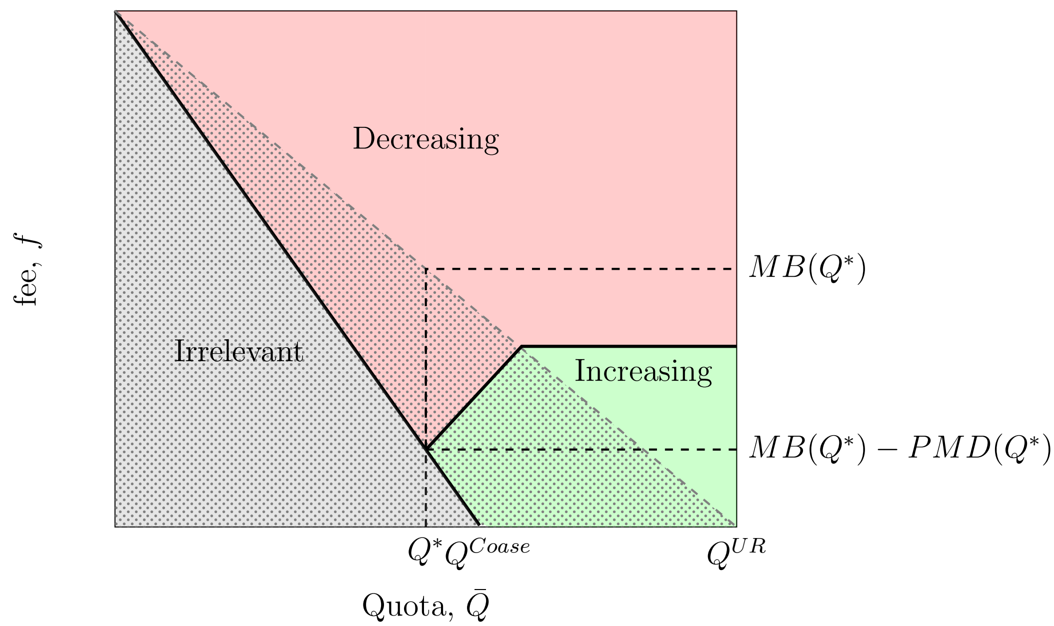
First, the welfare-irrelevant region shifts inward and pivots so that the frontier is steeper. This inward shift holds in general, not purely for the linear case depicted (Appendix B). Intuitively, increasing β raises PMD, shifting Q^{Coase} closer to the efficient quantity and reducing the minimum subsidy necessary to stimulate Coasean provision at a given $\bar{Q}$. The effect is to reduce the range of $(\bar{Q}, f)$ values that have no effect on welfare, while enlarging the range of policy instrument values that lead to either increases or losses in welfare from Coasean provision.

Second, reducing free-riding lowers the quantity of resource use in the purely Coasean equlibrium, Q^{Coase}. This increases the range of quota levels for which $f_0 = 0$, so that the presence of even a negligible fee induces welfare-improving Coasean provision for quotas above Q^{Coase}. Furthermore, the decrease in free-riding shifts a set of relatively stringent quota values between Q^* and Q^{Coase} out of the irrelevant set and into the increasing set when paired with moderate fee levels.

Third, reducing free-riding (increasing β) lowers the upper frontier of the welfare-increasing CP region. In particular, the origination point on the f axis falls by the difference between $PMD(Q^*)$ evaluated at the two β values. Comparative statics in the general case demonstrate that the upper bound on the increasing welfare region shifts uniformly downward as β increases; in other words, the slope of the frontier is not a function of β itself within the two regions in which either the quota or the fee binds (Appendix B).

The result of these changes is that f_1 uniformly falls with the increase in β, and the region

Figure 6: Welfare Implications of Coasean Provision (low free riding)



Notes: This figure depicts the welfare effects of Coasean provision given levels of the quota and fee with relatively low levels of free riding ($\beta = 0.7$).

over which it increases in $\bar{Q}$ expands. Intuitively, the reduction in free-riding causes a lower quantity under CP to a given fee than under higher free-riding. This greater behavioral response to the implicit subsidy provided by the fee magnifies the "overshoot" problem associated with higher fees, reducing the ceiling on the welfare-improving values of f. In general, decreases (increases) in free-riding hamper the ability of regulators to charge higher fees— perhaps to satisfy revenue-raising objectives—and still improve welfare under Coasean provision. Indeed, as Figure 6 shows, the maximum possible welfare-improving fee may be much lower than the first-best fee in the no-CP case ($MB(Q^*)$).

Proposition 4. *Increasing β (i.e. reducing free-riding) strongly reduces f_1 and weakly reduces f_0 for all $\bar{Q} \geq Q^*$.*

The overall effect of these changes on the range of welfare-improving fees $f \in (f_0, f_1)$ for a given $\bar{Q}$ depends on the value of $\bar{Q}$ and the magnitude of the change in β. For relatively large quota values, raising β unequivocally *reduces* the welfare-improving range of fees, whereas for some quota values nearer to Q^* the range can increase due to the fact that higher β values make previously irrelevant fee values relevant and able to support welfare-improving

CP.

3.3.2. The effect of transaction costs

There are many potential transaction costs that may serve as a form of "friction" to the realization of Coasean provision on top of a regulatory system. One of these, that of free-riding and costly collective action to actualize the demand for environmental quality, has been addressed already through the analysis of β. Environmental NGOs can play an important role in raising β through the use of effective outreach and fundraising operations, including those targeted specifically towards Coasean provision, in lieu of or complementary with more traditional lobbying or litigation approaches. Governments can play an important role as well by signaling their willingness to honor Coasean bargains and by clearly defining property rights, thereby potentially inducing would-be actors into the market. Ultimately, however, we argue that these actions to reduce the transaction costs of informing and organizing diffuse environmental "buyers" into a unified whole can be viewed as shifts in β, so that the comparative statics of these transitions are much as we described in the previous section.

In addition to the costs of aggregating preferences to realize the demand for environmental quality there may be significant transaction costs associated with negotiating, monitoring and enforcement of Coasean bargains themselves. So far these costs have been treated as negligible or as proportional to the overall size of the Coasean provision (i.e. reduced to a change in β). However, not all transaction costs may conform to these assumptions. Many transaction costs may take on a lumpy, or fixed character by bearing no direct relation to the magnitude of Coasean provision. For example, many of the costs of locating, negotiating, and contracting with resources users or polluters may be the same regardless of whether the scale of the contract is large or small in nature. Furthermore, it is plausible that the NGO or other entity representing conservation interests must bear these costs and therefore will consider whether the resulting Coasean bargain generates *private* net benefits to those that contribute to the public good after shouldering the transaction cost. In this case, it possible, even likely, that some previously feasible, and even net welfare-improving bargains will no longer be attractive to the Coasean "buyer."

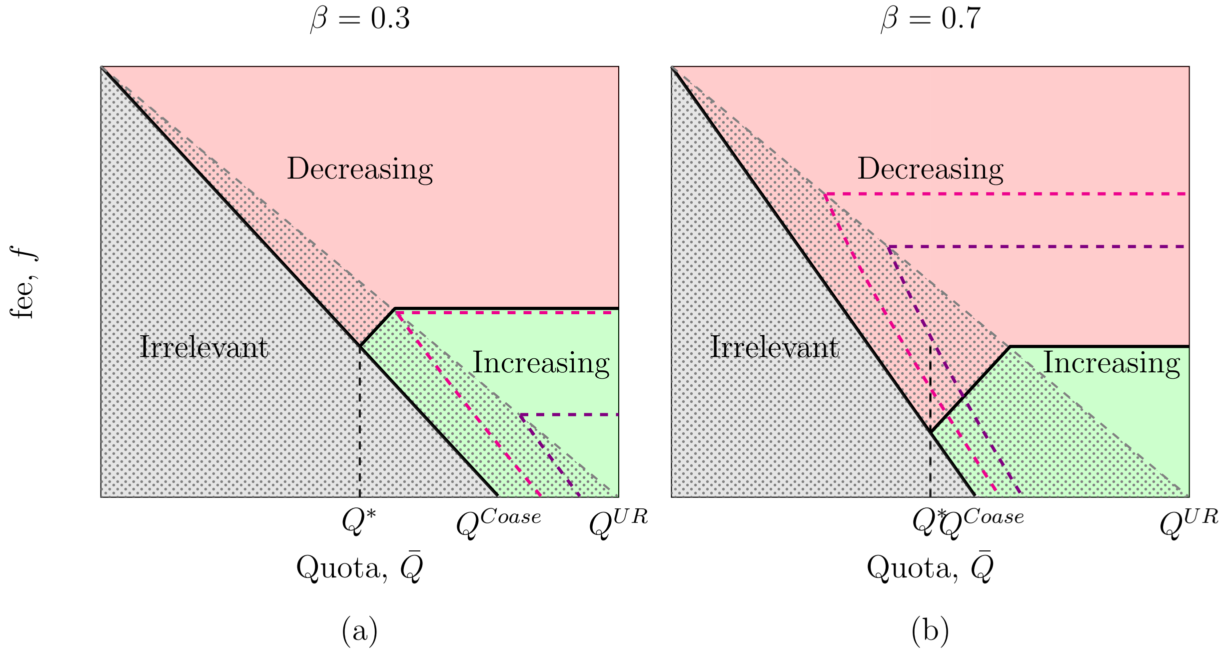
To consider this possibility, we solve for values of $(\bar{Q}, f)$ that generate positive private benefits to conservation buyers by integrating under the PMD curve between the regulatory status-quo Q and the CP equilibrium value. If this change in welfare exceeds the specified fixed transaction cost, then we deem the CP equilibrium feasible.

Figure 7 presents the feasible regions for the same two values of β examined before, but under two values of transaction costs: one relatively low, and another that is twice as large. These feasible boundaries consist of two parts. The horizontal upper bound on fees holds in the region where fees are binding in the regulatory equilibrium and represents the maximum fee consistent with a Coasean bargain given the level of fixed transaction costs. A higher fee reduces the value of $\hat{Q}$ in the CP equilibrium, but in doing so it also reduces the baseline Q under regulation. Since lower resource use comes at reduced private marginal damage, the increase of fees above the threshold value reduces the total private benefits of the Coasean bargain to the buyer such that it no longer compensates for its transaction costs. The downward-sloping portion of the feasibility frontier occurs when quota binds in the pre-CP baseline. Starting from a combination of $(\bar{Q}, f\ )$ that just cause the ENGO to "break even", decreasing the quota requires a compensating increase in the fee to increase private benefits after CP in order for the deal to "pay for itself."

Figure 7 reveals an important property of how the feasibility regions for CP vary under levels of free-riding. Under relatively low levels of free-riding (panel b) the feasibility regions are relatively expansive. This reflects the fact that the area under the PMD curve between any two quantities will be higher when free-riding is low such thatt NGOs have relatively "deep pockets." Nevertheless, the presence of fixed transaction costs can rule out many instances of Coasean provision that would have otherwise occurred in a world without the transaction cost. In particular, CP equlibria under particularly high fees—high Pigouvian fee equlibria, given that fees are binding in these cases—are excluded by transaction costs. Interestingly, these CP equilibria are also welfare-reducing relative to the regulatory status-quo. However, fixed transaction costs can also make infeasible otherwise welfare-improving bargains in quota-binding cases where the quota is relatively stringent but with relatively low to moderate fees.

Moving to cases of high free-riding (Figure 7, panel a) we find a similar pattern to the low free-riding case. However, the magnitude of the $(\bar{Q}, f)$ region in which CP bargains are

Figure 7: Welfare Implications of Coasean Provision with Fixed Transaction Costs and Different Levels of Free-riding.



Notes: Dotted lines enclose regions where Coasean provision is feasible under low (magenta) and high (violet) fixed transaction costs. The assumption for transaction costs is that "buyer pays," so that the benefits from CP to the non free-riders must exceed the transaction costs.

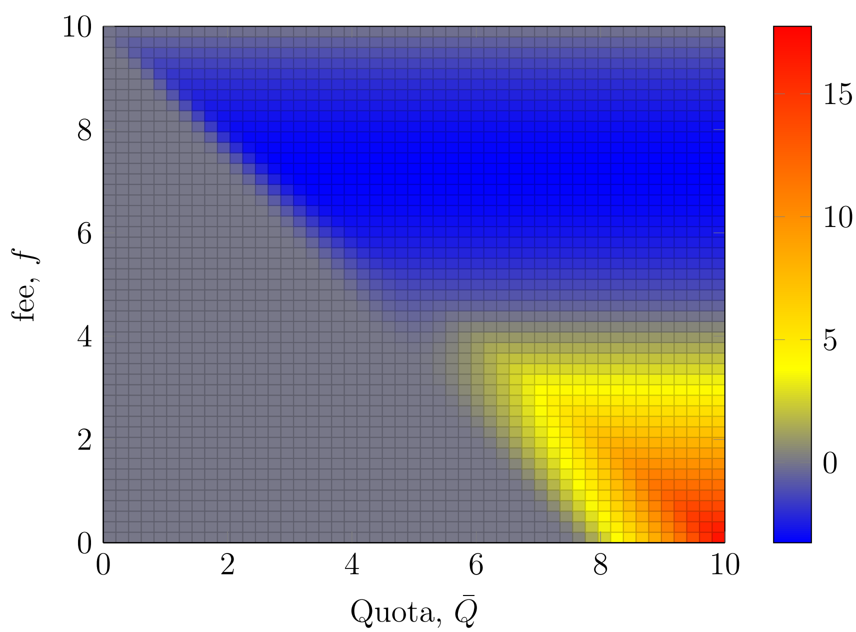
now feasible has shrunk substantially. Importantly, this region shrinks toward the bottom right of the figure – situations with relatively lax quotas and low fees. In short, as free-riding increases and transaction costs increase, Coasean bargaining becomes unattractive except in cases where the regulatory equilibria closely resemble the unregulated outcome.

Importantly, under the assumption that the MB curve is uniformly downward sloping and the MD and PMD curves all are upward sloping in Q, it is straightforward to see that the *total* private benefits of CP, the area under the PMD curve between the baseline provision under regulation and $\hat{Q}$, increase at a decreasing rate in the quantity of CP and will be at its largest when the pre-CP equilbrium is lax. The result of this property is that the private and social welfare gains from CP are relatively well-aligned (Figure 8). Therefore, introducing fixed transaction costs of a moderate magnitude will tend to preserve the highest social welfare equilibria as feasible, while excluding the most welfare destroying equlibria.

Taken together, these patterns suggest fixed transaction costs play an interesting double role. First, moderately high transaction costs may actually play a valuable function in a world where CP is allowed by preventing CP equlibria that would actually be welfare-reducing. This is clearly seen in the panel b of Figure 7 where the level of transaction costs associated with the magenta boundary exclude *all* trades that would result in negative changes in welfare. However, this property comes at the cost of excluding relatively modest welfare gains from CP in cases where quotas are relatively stringent and fees are sufficiently low that the quotas are binding under regulation.

Finally, as demonstrated in Figure 7, the presence of free-riding behavior interacts in important and policy-relevant ways with the level of transaction costs. A transaction cost that would tend to permit most welfare-improving CP under even moderately stringent regulation with low free-riding (panel b, purple feasibility region) can exclude the possibility of Coasean provision in all but the most lax regulatory environments with high free-riding (panel a, purple feasibility region). Of course, the converse of this observation is that "bad trades" that were excluded under a high free-riding regime may actually become feasible and occur if governments or ENGOs innovate to reduce free riding—even if the regulatory environment remains unchanged.

Figure 8: Additional Social Welfare from Coasean Provision



Notes: This is welfare when $\beta = 0.3$.

3.4. Summary and Policy Implications

Synthesizing these theoretical findings provides us with a few straightforward insights concerning when allowing Coasean provisioning, or reducing the transaction costs to Coasean provisioning, is likely be welfare improving.

First, if baseline environmental policy is stringent relative to what would achieve social efficiency (i.e. $\bar{Q} \leq Q^*$ or $f \geq MB(Q^*)$) then there is *no* potential for Coasean provision to improve welfare. At best, CP will simply fail to materialize and therefore have no welfare impact at all. Importantly, the potential for doing harm by allowing CP differs substantially across fee and quota instruments. In the case of a quota system without a fee, lowering $\bar{Q}$ below Q^* has no impact on CP whatsoever, so that allowing CP does no harm. However, if resource use is regulated by fees alone, then allowing voluntary provision when the fee is already set at or above the efficient level can only lower welfare due to the subsidy effect of the fee on Coasean provision. In the dual instrument case, knowing whether allowing CP will prove harmful to welfare vs. merely irrelevant hinges critically on whether the fee is sufficiently large to induce Coasean provision or not, given the quota level. If the fee is greater than the gap between WTA and private WTP, then allowing CP will harm welfare. Knowing whether this is the case or not in practice could be difficult. However, it is clear based upon the previous arguments for fee-only instruments as well as graphical intuition (Fig. 5) that if the fee is binding rather than the quota prior to allowing CP (i.e. changes in quota have no effect on quantity, whereas changes in fees do), then welfare will be harmed.

Second, if quotas are high relative to the efficient level ($\bar{Q} > Q^*$) and fees are sufficiently small, where "sufficiently small" is defined as less than the gap between the marginal WTA of resource users (before subsidization) and the private marginal damages to ENGOs at Q^*—$MB(Q^*) - PMD(Q^*)$, the fee that will induce efficient Q under Coasean provision—then allowing Coasean provision will definitely do no harm and may increase welfare. In this case, if the quota is set above the level that would be achieved by Coasean bargaining alone, Q^{Coase}, then allowing Coasean provisioning on top of regulation will unequivocally increase welfare. If, on the other hand, $\bar{Q} < Q^{Coase}$ then whether allowing CP will increase welare or not will depend on whether the fee is sufficiently large to induce voluntary transactions in

Q. The degree of free-riding reflected in ENGOs' demands will be critical to whether or not a particular fee/quota combination that meets these overall criteria will improve welfare vs. simply prove irrelevant. Low free-riding increases the range of $\bar{Q}$ that will improve welfare at zero or very low fee levels relative to high free-riding cases. However, lowering free-riding also lowers the maximum fee $MB(Q^*) - PMD(Q^*)$ than be guaranteed to "do no harm" (Figure 7).

Third, if quotas are above the efficient level and fees exceed the level required to induce efficient Coasean provision ($f > MB(Q^*) - PMD(Q^*)$) then it is far less clear *a priori* whether allowing Coasean provision will enhance welfare or not. The one thing that is certain is that positive CP will occur at these fee levels and so welfare will not be unaffected by Coasean provision.[7] In general, the shape of the upper bound of the upper bound of the increasing welfare region, f_1, is a function of the higher-order derivatives of the MB, MD, and PMD functions, particularly the extent of free-riding. This finding suggests that it may be difficult to determine *a priori* whether allowing Coasean provision will improve welfare in settings with non-negligible fees without significant research efforts to understand the underlying functional relationships.

Fourth, understanding the nature and magnitude of transaction costs facing would-be Coasean bargains is critical for establishing the welfare implications of allowing voluntary provision on top of regulation. In particular, if transaction costs are fixed in nature and paid by the Coasean buyer, then many trades will simply be blocked. While this may have the unfortunate consequence of preventing welfare-improving CP, the effects of transaction costs are not completely negative since the $(\bar{Q}, f)$ pairs that remain viable for Coasean provision become increasingly likely to be in the welfare-improving range as transaction costs rise. Indeed, if transaction costs reach a critical level (where this threshold decreases in the extent of free-riding) then allowing Coasean provision will either increase welfare or prove irrelevant. Therefore, transaction costs may prevent beneficial Coasean provision, but they also play a protective role by selecting against "bad trades."

[7] Technically this is only true if fixed transaction costs are zero. Positive transaction costs could yield welfare-irrelevant outcomes.

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Appendix A. Characterization of CP equilibria

The CP equilibria can be qualitatively characterized based on the ordering of four quantities: Q^*, the efficient level of resource use; $\bar{Q}$, the quota or cap under the policy; Q_f, defined by $MB(Q_f) - f = 0$, the quantity dictated by the fee under the regulation assuming the fee were binding; and $\hat{Q}$, the level of resource use under Coasean provision.

Given these variables, there are potentially **24** different cases to consider. However, only **12** of these are actually possible since any scenario where $Q_f < \hat{Q}$ cannot occur in practice. To see why, observe that Q_f is defined as the intersection of the $MB - f$ curve and the x axis, while $\hat{Q}$ is the intersection of the $MB - f$ curve and the upward-sloping, strictly positive PMD curve. Hence, it must be the case that PMD intersects $MB - f$ at a lower value of Q than Q_f. This rules out **12** of the **24** possible orderings of the four values in which $Q_f < \hat{Q}$.

Moving on to the remaining 12 cases, they can be classified as follows.

Welfare-irrelevant. If $\bar{Q}$ is set so stringently that it lies below $\hat{Q}$, then allowing CP is irrelevant because there would be no trade—extraction is already capped *below* the level that extractors and NGOs would bargain to. There are **4** cases where this occurs.

- $Q^* < \bar{Q} < \hat{Q} < Q_f$
- $\bar{Q} < Q^* < \hat{Q} < Q_f$
- $\bar{Q} < \hat{Q} < Q^* < Q_f$
- $\bar{Q} < \hat{Q} < Q_f < Q^*$

Welfare reducing. The next set of cases are those where the binding quota or fee instrument is stringent relative to the optimal level ($\bar{Q} < Q^*$ and/or $Q_f < Q^*$). These **4** cases, differentiated by whether the quota or fee binds in regulatory equilibrium without CP, are:

1. $\hat{Q} < \bar{Q} < Q^* < Q_f$ (Quota binds)
2. $\hat{Q} < \bar{Q} < Q_f < Q^*$ (Quota binds)
3. $\hat{Q} < Q_f < Q^* < \bar{Q}$ (Fee binds)

4. $\hat{Q} < Q_f < \bar{Q} < Q^*$ (Fee binds)

Welfare improving. The next set of **2** cases are those where CP results in intermediate levels of extraction that are lower than under regulation alone, but higher than the optimal level.

5. $Q^* < \hat{Q} < \bar{Q} < Q_f$ (Quota binds)
6. $Q^* < \hat{Q} < Q_f < \bar{Q}$ (Fee binds)

Welfare ambiguous. Finally, there are **2** cases in which the CP equilibrium is at a *lower* level of extraction than the optimal level while the no-CP regulatory equilibrium is above Q^*.

7. $\hat{Q} < Q^* < \bar{Q} < Q_f$ (Quota binds)
8. $\hat{Q} < Q^* < Q_f < \bar{Q}$ (Fee binds)

Appendix B. Comparative statics

This section examines the case for generalized MB, MD, and PMD functions as well as for the linear case. In the linear case notation is defined as follows.

$$MB(Q) = a - bQ$$
$$MD(Q) = c + dQ$$
$$PMD(Q) = \beta MD(Q)$$

Properties of the welfare-irrelevance frontier

If $f < MB(\bar{Q}) - PMD(\bar{Q})$ then Coasean provision will not occur. Defining the "irrelevance frontier" as $f = MB(\bar{Q}) - PMD(\bar{Q})$ the shape of the frontier is as follows:

$$\frac{\partial f}{\partial \bar{Q}} = MB' - PMD' < 0$$
$$\frac{\partial^2 f}{\partial \bar{Q}^2} = MB'' - PMD''$$
$$\frac{\partial f}{\partial \beta} = -MD' < 0$$
$$\frac{\partial^2 f}{\partial \bar{Q} \partial \beta} = -MD''$$

The frontier is downward sloping given the slopes of the MB and PMD functions. The curvature of the frontier depends on the curvature of these functions. If $MB'' < 0$ and $PMD'' > 0$ then $\frac{\partial^2 f}{\partial \bar{Q}^2} < 0$ whereas if $MB'' > 0$ and $PMD'' < 0$ then $\frac{\partial^2 f}{\partial \bar{Q}^2} > 0$.

Reducing free riding (increasing β) shifts the frontier downward. Whether this translation is uniform or not depends on the curvature of the MD function.

Considering the special case of linearity, then $f = (a - \beta c) - (b + \beta d)\bar{Q}$.

$$\frac{\partial f}{\partial \bar{Q}} = -(b + \beta d) < 0$$
$$\frac{\partial f}{\partial \beta} = -(c + d\bar{Q}) < 0$$
$$\frac{\partial^2 f}{\partial \bar{Q} \partial \beta} = -d < 0''$$

Under linearity, the irrelevance frontier is linear in $\bar{Q}$. In addition to shipping downward as β increases, the slope also rotates inward, becoming steeper in $\bar{Q}$.

Properties of the increasing/decreasing welfare boundary

In general, the change in welfare from Coasean provision relative to the regulatory outcome is.

$$\Delta W^{CP} = \int_{\hat{Q}}^{min(\bar{Q}, Q_f)} MB(Q) - MD(Q)\, dQ$$

To explore the shape of the $(\bar{Q}, f)$ frontier between increasing and decreasing welfare under CP, define $\Omega(\bar{Q}, f) \equiv \Delta W^{CP} = 0$. This provides an implicit function revealing the locus of $(\bar{Q}, f)$ such that welfare after Coasean provisioning is unchanged. Let us assume, at first, that we are in a region where $\bar{Q}$ binds. Then, by Leibniz rule and the implicit function theorem we obtain:

$$\left.\frac{df}{d\bar{Q}}\right|_{\Delta W=0} = -\frac{\frac{\partial \Omega}{\partial \bar{Q}}}{\frac{\partial \Omega}{\partial f}} = \frac{MB(\bar{Q}) - MD(\bar{Q})}{\left(MB(\hat{Q}) - MD(\hat{Q})\right)\frac{d\hat{Q}}{df}} \tag{B.1}$$

The numerator of (B.1) is > 0 given the slope properties of MB and MD and since $\bar{Q} > Q^*$ in the "welfare ambiguous" transition region between positive and negative welfare changes. Similarly the first term in the denominator is negative since $\hat{Q} < Q^*$ in this region as well. Given that $\frac{d\hat{Q}}{df} < 0$ (Proposition 2), the overall denominator is positive. Therefore, $\left.\frac{df}{d\bar{Q}}\right|_{\Delta W=0} > 0$ when $\bar{Q}$ binds.

Now, consider the case where the fee binds in the no-CP regulatory equilibrium. In this case $min(\bar{Q}, Q_f) = Q_f$. However, this implies that the upper limit of integration for $\Omega(\bar{Q}, f)$ is now no longer a function of $\bar{Q}$. Therefore, the same mathematical logic as for (B.1) shows that $\frac{d\Omega}{dQ} = 0$ so that $\left.\frac{df}{d\bar{Q}}\right|_{\Delta W=0} = 0$ when the fee binds. Intuitively, if the quota does not bind in the no-CP equilibrium then marginal changes to it are immaterial to the baseline Q. Since the fee binds in the CP equilibrium (outside of the welfare irrelevance region), the quota has no impact on either the baseline or the CP equlibrium Q and therefore no impact on the change in welfare under CP.

Now let us consider how the increasing/decreasing frontier changes with β. Note that the upper limit of integration for $\Omega(\bar{Q}, f)$ is not a function of β, regardless of whether the fee or quota binds. Therefore, the following derivative holds across the entire frontier.

$$\left.\frac{df}{d\beta}\right|_{\Delta W=0} = -\frac{\frac{\partial \Omega}{\partial \beta}}{\frac{\partial \Omega}{\partial f}} = \frac{-\left(MB(\hat{Q}) - MD(\hat{Q})\right)\frac{d\hat{Q}}{d\beta}}{\left(MB(\hat{Q}) - MD(\hat{Q})\right)\frac{d\hat{Q}}{df}} = -\frac{\frac{d\hat{Q}}{d\beta}}{\frac{d\hat{Q}}{df}} \tag{B.2}$$

Both the numerator and denominator derivatives are negative; therefore, $\left.\frac{df}{d\beta}\right|_{\Delta W=0} < 0$. An increase in β (a decrease in free-riding) shifts the frontier between increasing and decreasing welfare downward.

Finally, it is straightforward to demonstrate that the slope, $\frac{df}{d\bar{Q}}\Big|_{\Delta W=0}$, of the increasing/decreasing welfare boundary is unchanged in β. Differentiating (B.2) with respect to $\bar{Q}$ yields:

$$\frac{d^2 f}{d\bar{Q}d\beta}\bigg|_{\Delta W=0} = -\frac{d^2\hat{Q}}{d\bar{Q}d\beta}\left(\frac{d\hat{Q}}{df}\right)^{-1} + \frac{d\hat{Q}}{d\beta}\left(\frac{d\hat{Q}}{df}\right)^{2}\frac{d^2\hat{Q}}{d\bar{Q}df}$$

Each of the cross-partial derivatives on the RHS of this expression are zero due to the fact that the CP equilibrium quantity, $\bar{Q}$, is not a function of $\bar{Q}$ in this region of $(\bar{Q}, f)$ space. Therefore, $\frac{d^2 f}{d\bar{Q}d\beta}\Big|_{\Delta W=0} = 0$.

In the linear case, we can write ΔW^{CP} where $\bar{Q}$ binds as.

$$\Delta W^{CP} = (a-c)\bar{Q} - \frac{1}{2}(b+d)\bar{Q}^2 - (a-c)\hat{Q} + \frac{1}{2}(b+d)\hat{Q}^2 \tag{B.3}$$

To find the slope of the increasing/decreasing welfare frontier, define $\Omega(\bar{Q}, f) \equiv \Delta W^{CP} = 0$ and utilize the implicit function theorem. After some simple algebra, this yields:

$$\frac{df}{d\bar{Q}} = -\frac{\frac{\partial\Omega}{\partial\bar{Q}}}{\frac{\partial\Omega}{\partial f}} = -(b+\beta d)\cdot\frac{Q^* - \bar{Q}}{Q^* - \hat{Q}} \tag{B.4}$$

where $\hat{Q} = \frac{a-\beta c-f}{b+\beta d}$.

To find an explicit solution for the frontier, define $\phi(\bar{Q}) = (a-c)\bar{Q} - \frac{1}{2}(b+d)\bar{Q}^2$. Then $\Omega(\bar{Q}, f)$ can be defined as

$$\frac{1}{2}(b+d)\hat{Q}^2 - (a-c)\hat{Q} + \phi(\bar{Q}) = 0 \tag{B.5}$$

We can then solve for the roots of this quadratic equation in $\hat{Q}$.

$$\hat{Q}(\bar{Q}) = \frac{(a-c) \pm \sqrt{(a-c)^2 - 2(b+d)\phi(\bar{Q})}}{b+d} \tag{B.6}$$

where only one of these two roots will be sensible.

Knowing $\hat{Q} = \frac{a-\beta c-f}{b+\beta d}$ we can solve for f as a function of $\hat{Q}$, $f = (a-\beta c) - (b+\beta d)\hat{Q}$ and then plug our expression for $\hat{Q}(\bar{Q})$ into this equation.

$$f(\bar{Q}) = (a - \beta c) - (b + \beta d) \cdot \hat{Q}(\bar{Q}) \tag{B.7}$$